65 $y' = x - \frac{1}{x^2}$

$y' = x - x^{-2}$

a) $(0,\infty)$

$y = \frac{x^2}{2} - \frac{x^{-1}}{-1} + C$

$y = \frac{x^2}{2} + \frac{1}{x} + C$

$2 = \frac{1}{2} + 1 + C$

$\frac{1}{2} = C$

$x=-1 \quad y=1$

b) $1 = \frac{1}{2} - \frac{1}{1} + C$

$\frac{3}{2} = C$

c) $y = \begin{cases} \frac{x^2}{2} + \frac{1}{x} + C_1, & x<0 \\ \frac{x^2}{2} + \frac{1}{x} + C_2, & x>0 \end{cases}$

Dec 9-12:41 PM

6.2 u- substitutions

memorize basic formulas on p 332

$$\int u^n \, du = \frac{u^{n+1}}{n+1} + C$$

why "u" instead of "x"?

"u" is a function of x

Dec 9-1:19 PM

$$\int (\sin x)^3 \cos x \, dx = \int u^3 \, du$$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$du = \cos x \cdot dx$

$= \frac{u^4}{4} + C$

$= \frac{(\sin x)^4}{4} + C$

Dec 9-1:25 PM

$$\int e^{\tan x} \sec^2 x \, dx = \int e^u \, du$$

$u = \tan x$

$\frac{du}{dx} = \sec^2 x$

$du = \sec^2 x \, dx$

$= e^u + C$

$= e^{\tan x} + C$

Dec 9-1:33 PM

$$\int \cot x \, dx$$

$$\int \frac{\cos x}{\sin x} \, dx = \int \frac{du}{u} = \int \frac{1}{u} \, du$$

$u = \sin x$

$du = \cos x \, dx$

$= \ln|u| + c$

$= \ln|\sin x| + c$

Dec 9-1:36 PM

$$\int \cot(7x) \, dx$$

$$\frac{1}{7}\int \frac{7\cos(7x)}{\sin(7x)} \, dx = \frac{1}{7}\int \frac{du}{u}$$

$u = \sin(7x)$

$\frac{du}{dx} = 7\cos(7x)$

$du = 7\cos(7x)\,dx$

$= \frac{1}{7}\ln|u| + c$

$= \frac{1}{7}\ln|\sin(7x)| + c$

Dec 9-1:43 PM

$$\frac{1}{2}\int 2x\sqrt{5+x^2}\,dx = \frac{1}{2}\int \sqrt{u}\,du$$

$u = 5+x^2$

$du = 2x\,dx$

$$= \frac{1}{2}\int u^{\frac{1}{2}}\,du$$

$$= \frac{1}{2}\,\frac{u^{3/2}}{3/2} + C$$

$$= \frac{1}{2}\cdot\frac{2}{3}\left(5+x^2\right)^{3/2} + C$$

$$= \frac{\left(5+x^2\right)^{3/2}}{3} + C$$

Dec 9-1:48 PM

$$-\frac{1}{2}\int_0^1 \frac{-2x}{4-x^2}\,dx = -\frac{1}{2}\int_4^3 \frac{du}{u} = \frac{1}{2}\int_3^4 \frac{1}{u}\,du$$

✓ $u = 4-x^2$

$\frac{du}{dx} = -2x$

$du = -2x\,dx$

$x=0 \quad u=4$

$x=1 \quad u=3$

$$= \frac{1}{2}\ln|u|\Big|_3^4$$

$$= \frac{1}{2}\ln 4 - \frac{1}{2}\ln 3$$

$$= \frac{1}{2}\left(\ln 4 - \ln 3\right)$$

$\frac{1}{2}\ln\frac{4}{3} = \ln\left(\frac{4}{3}\right)^{\frac{1}{2}}$

$$= \ln\sqrt{\frac{4}{3}}$$

Dec 9-1:54 PM